

# Is the cyclic prefix necessary?

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## Abstract

We show that one can do away with the cyclic prefix (CP) for SC-FDE and OFDM at the cost of a moderate increase in the complexity of a DFT-based receiver. Such an approach effectively deals with the decrease in the number of channel uses due to the introduction of the CP. It is shown that the SINR for SC-FDE remains the same asymptotically with the proposed receiver without CP as that of the conventional receiver with CP. The results are shown for  $N_t$  transmit antennas and  $N_r$  receive antennas where  $N_r \geq N_t$ .

## I. INTRODUCTION

Inter-symbol Interference (ISI) introduced by the time-varying multi-path channel is one of the main limiting factors for high speed data communications. Discrete Fourier Transform (DFT) based receiver used in SC-FDE (single-carrier frequency-domain equalization) and OFDM (orthogonal frequency-division multiplexing) offer a low complexity alternative to deal with this problem (see for example [1], [2], [3] and references therein).

Fundamentally, these techniques rely on the redundancy in the form of CP that with appropriately chosen length makes the linear convolution introduced by the physical channel look like a circular convolution that can be dealt well with DFT or Inverse DFT (IDFT). This redundancy however results in decrease of the number of channel uses available for signal transmission.

We will show that without CP and using a receiver that is designed to work with cyclic prefix, one suffers from an interference component at the start and the end of the DFT frame. This edge effect decays exponentially as one moves inside the DFT frame unless the poles of a certain inverse filter are on the unit circle, in which case, the interference is present for the entire frame.

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We show that it is guaranteed that for the MMSE SC-FDE, the poles will never lie on the unit circle and hence the edge effect will truly be at the edges assuming that the DFT frame is large enough so that the symbols at the interior of the frame are unaffected due to the exponential decay. Hence by discarding the symbols at the edges and re-processing the discarded symbols by putting them in the interior of another frame, one can obtain a performance that can made as close as possible to the one obtained with CP.

We define the notation as used throughout this paper:

- $\{y_m\}$  denotes a finite length sequence of vectors indexed by  $m$  whose length is specified in the context, where  $y_m$  is a  $N_t \times 1$  vector whose  $k$ th element is denoted by  $y_{m,k}$ .
- DFT of a  $N$ -point vector-sequence is denoted by  $\{\tilde{y}_n\} = \text{DFT}\{y_m\}$  and is given by

$$\tilde{y}_n = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} y_m e^{-j2\pi mn/N}. \quad (1)$$

Note that DFT is element-wise and one can write (1) as

$$\tilde{y}_{n,k} = \frac{1}{\sqrt{N}} \sum_{m=0}^{N-1} y_{m,k} e^{-j2\pi mn/N}. \quad (2)$$

- Inverse DFT (IDFT) of a  $N$ -point vector-sequence is denoted by  $\{y_m\} = \text{IDFT}\{\tilde{y}_n\}$  and is given by

$$y_m = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{y}_n e^{j2\pi mn/N}. \quad (3)$$

- $\mathcal{CN}(0, \sigma^2)$  denotes a circularly symmetric complex Gaussian random variable with zero mean and variance of  $\sigma^2$ .
- Let  $x_m = \{h_m \otimes u_m\} \triangleq \sum_{q=0}^{N-1} h_q u_{(m-q) \bmod N}$  denote the circular convolution of the two  $N$ -point sequences  $\{h_m\}$  and  $\{u_m\}$ , where  $h_m$  is a  $N_r \times N_t$  matrix and  $u_m$  is a  $N_t \times 1$  vector. Note that if  $\{\tilde{x}_n\} = \text{DFT}\{x_m\}$ ,  $\{\tilde{h}_n\} = \text{DFT}\{h_m\}$ , and  $\{\tilde{u}_n\} = \text{DFT}\{u_m\}$ , then  $\tilde{x}_n = \sqrt{N} \tilde{h}_n \tilde{u}_n$ .
- $u^*$  denotes the conjugate of  $u$ .
- $\|A\|_\infty$  denotes the  $L$ -infinity norm of matrix  $A$  given by  $\|A\|_\infty = \max_{i,j} |A_{i,j}|$ .
- $\mathbb{E}\{\cdot\}$  denotes the expectation.
- $x^\dagger$  denotes the conjugate transpose of  $x$ .

## II. SYSTEM MODEL

Let  $\{u_m\}$ ,  $m = -\infty, \dots, \infty$ ,  $u_m \in \mathcal{C}^{N_t}$  be the input to a  $L$ -tap channel given by

$$\underline{\mathbf{h}} = [h_0, \dots, h_{L-1}],$$

where  $h_i$ 's are  $N_r \times N_t$  matrices. If  $x_m$  and  $\nu_m$  denote the channel output and noise respectively at time instant  $m$ , then the system model is given by

$$x_m = \sum_{l=0}^{L-1} h_l u_{m-l} + \nu_m. \quad (4)$$

We will assume that the channel is perfectly known at the receiver, is quasi-static that remains constant over a frame and changes independently from one frame to another, whose elements are  $\mathcal{CN}(0, 1)$ , and the channel random process is spatially and temporally white,  $\nu_m$  is i.i.d. whose elements are uncorrelated and each element is  $\mathcal{CN}(0, N_0)$ , and without loss of generality that the average transmitted power is unity i.e.  $\mathbb{E}\{\|u_m\|^2\} = 1$ .

In what follows, we consider OFDM, CP SC-FDE, CP-less SC-FDE, and CP-less OFDM with the CP present in the first two techniques.

We will divide the transmitted symbols into frames of length  $N + C$  each, where  $N$  denotes the number of symbols carrying information and  $C$  denotes the number of redundant symbols due to CP. It is well-known that for  $C \geq L - 1$ , the linear convolution will be the same as the circular convolution after discarding the first  $C$  samples of the received signal. Let the information carrying symbols be denoted by

$$\underline{\tilde{\mathbf{y}}} = [\tilde{y}_0, \dots, \tilde{y}_{N-1}]. \quad (5)$$

For OFDM only, these symbols undergo an additional transformation by using the IDFT as

$$\{y_m\} = \text{IDFT}\{\tilde{y}_n\}. \quad (6)$$

For CP SC-FDE and CP-less SC-FDE,

$$y_m = \tilde{y}_m, \quad \forall \quad m. \quad (7)$$

The transmitted frame of size  $N_t \times (N + C)$  is transmitted during the time instants from  $-C, -C + 1, \dots, 0, 1, \dots, N - 1$  and is given by

$$\underline{\mathbf{u}} = [\overbrace{y_{N-C}, y_{N-C+1}, \dots, y_{N-1}}^{\text{Cyclic Prefix}, C}, \overbrace{y_0, y_1, \dots, y_{N-1}}^{\text{Data}, N}]. \quad (8)$$

For the CP-less case,  $C = 0$  and hence the transmitted signal in time instants  $-C, \dots, N-1$  is given by

$$\mathbf{u} = [\overbrace{w_{N-C}, \dots, w_{N-1}}^{\text{From previous frame, } C}, \overbrace{y_0, \dots, y_{N-1}}^{\text{Data, } N}], \quad (9)$$

where  $w_m$ 's are the transmitted vectors from the previous frame.

### III. RECEIVER

The channel output is recorded at the time instants  $m = 0, \dots, N-1$  and we take the  $N$ -point DFT given by

$$\{\tilde{r}_n\} = \text{DFT}\{x_m\}. \quad (10)$$

#### A. OFDM Receiver

For the OFDM receiver, using (6) we have

$$\begin{aligned} \tilde{r}_n &= \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} x_q e^{-j2\pi qn/N} \\ &= \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} \left( \sum_{l=0}^{L-1} h_l u_{q-l} + \nu_q \right) e^{-j2\pi qn/N} \\ &= \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} \left( \sum_{l=0}^{L-1} h_l y_{(q-l) \bmod N} + \nu_q \right) e^{-j2\pi qn/N} \\ &= \sqrt{N} \tilde{h}_n \tilde{y}_n + \tilde{\nu}_n, \end{aligned} \quad (11)$$

where  $\{\tilde{\nu}_n\} = \text{DFT}\{\nu_q\}$  and  $\{\tilde{h}_n\} = \text{DFT}\{h_l\}$ . Since DFT is unitary,  $\{\tilde{\nu}_n\}$  has the same statistics as  $\{\nu_m\}$ .

#### B. CP SC-FDE Receiver

Following the analysis for the OFDM receiver, we arrive at (11). Note that unlike OFDM the information bearing signals for this case are  $y_m$ 's. We multiply (11) by a  $N_t \times N_r$  matrix

denoted by  $\tilde{g}_n$  and then take the IDFT to get

$$\begin{aligned}
\hat{y}_m &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{g}_n \tilde{r}_n e^{j2\pi mn/N} \\
&= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{g}_n \tilde{h}_n \tilde{y}_n e^{j2\pi mn/N} + \\
&\quad \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{g}_n \tilde{\nu}_n e^{j2\pi mn/N} \\
&\stackrel{a}{=} \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} p_k y_{(m-k) \bmod N} + \\
&\quad \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{g}_n \tilde{\nu}_n e^{j2\pi mn/N},
\end{aligned} \tag{12}$$

where in a,  $\tilde{p}_n = \tilde{g}_n \tilde{h}_n$ , and

$$\{p_m\} = \text{IDFT}\{\tilde{g}_n \tilde{h}_n\}. \tag{13}$$

Note that for the zero-forcing (ZF) SC-FDE,

$$\tilde{g}_n = \frac{1}{\sqrt{N}} (\tilde{h}_n^\dagger \tilde{h}_n)^{-1} \tilde{h}_n^\dagger, \tag{14}$$

and for the MMSE SC-FDE,

$$\tilde{g}_n = \sqrt{N} (N \tilde{h}_n^\dagger \tilde{h}_n + N_0 I)^{-1} \tilde{h}_n^\dagger. \tag{15}$$

We note that in (15), we are first doing the ZF or MMSE reception in the frequency domain and then taking the DFT. Since DFT is a unitary operation, it can be easily shown that ZF or MMSE in time domain can be written as a concatenation of ZF or MMSE respectively in frequency domain followed by DFT. This is shown in the appendix.

### C. CP-less SC-FDE

The receiver is kept the same as the CP SC-FDE except that one could vary the DFT frame size. In this case, the absence of CP doesn't make the linear convolution as a circular convolution and there is a spill-over of the signals from the previous frame causing interference. As we shall see below, for the MMSE case in particular, it is guaranteed that the interference levels will fall exponentially as one moves inside the frame from either of the two ends. Because of this phenomenon, one can discard the symbols at either ends of the frame and declare only the interior symbols of the frame as the equalized symbols. The discarded symbols at either

end can be equalized by putting them on the interior of a different frame. This indicates that having sliding and over-lapping frames (with the overlap determined by the rate of decay of the interference levels) will result in equalization of all symbols. It also follows that the additional interference due to lack of CP can be *controlled* by increasing the number of discarded symbols. Hence by increasing the receiver complexity, one can compensate for the absence of CP and also increase the effective number of channel uses as compared to the CP-based transmission.

We take the  $N$ -point DFT of the received sequence as in OFDM/CP SC-FDE to get

$$\begin{aligned}\tilde{r}_n &= \frac{1}{\sqrt{N}} \sum_{q=0}^{N-1} x_q e^{-j2\pi qn/N} \\ &= \sqrt{N} \tilde{h}_n \tilde{y}_n + \tilde{v}_n + \tilde{\kappa}_n,\end{aligned}\quad (16)$$

where

$$\{\tilde{\kappa}_n\} = \text{DFT} \left\{ \sum_{l=q+1}^{L-1} h_l (w_{N+q-l} - y_{N+q-l}) \right\}. \quad (17)$$

We note that the last summation term in (16) is the only difference between CP-less case and the schemes with CP. We multiply  $\tilde{r}_n$  in (16) by  $\tilde{g}_n$  and then take the IDFT to get

$$\begin{aligned}\hat{y}_m &= \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{g}_n \tilde{r}_n e^{j2\pi mn/N} \\ &= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} p_k y_{(m-k) \bmod N} + \\ &\quad \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \tilde{g}_n \tilde{v}_n e^{j2\pi mn/N} + \xi_m,\end{aligned}\quad (18)$$

where  $\tilde{p}_n = \tilde{g}_n \tilde{h}_n$ ,

$$\{p_m\} = \text{IDFT}\{\tilde{g}_n \tilde{h}_n\}, \quad (19)$$

$$\xi_m = \sum_{q=0}^{L-2} \sum_{l=q+1}^{L-1} \gamma_{(m-q) \bmod N} h_l (w_{N+q-l} - y_{N+q-l}), \quad (20)$$

and  $\{\gamma_m\} = \text{IDFT}\{\tilde{g}_n\}/\sqrt{N}$  i.e.

$$\gamma_m = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{g}_n e^{j2\pi mn/N}. \quad (21)$$

Let us assume that it is possible to choose a  $D > L - 1$  and  $D \leq m \leq N - D$ , such that  $\|\gamma_m\|_\infty < \epsilon$ , for any  $\epsilon > 0$ . Then for  $D \leq m \leq N - D$ , one can upper bound  $\|\xi_m\|$  as

$$|\xi_m| \leq \sum_{q=0}^{L-2} \sum_{l=q+1}^{L-1} N_r N_t \beta_1 \beta_2 \beta_3 \epsilon \quad (22)$$

$$\leq \frac{(L-2)(L-1)N_r N_t \beta_1 \beta_2 \beta_3 \epsilon}{2}, \quad (23)$$

where  $\beta_1 = \max_q \|\gamma_{(m-q) \bmod N}\|_\infty$ ,  $\beta_2 = \max_l \|h_l\|_\infty$  and  $\beta_3 = \|w_{N+q-l} - y_{N+q-l}\|_\infty$ .

Hence by choosing  $\epsilon$  small enough, one can make the interference term  $\xi_m$  that arises due to the absence of CP negligible. This makes the SINR of the symbols in the interior of the frame the same as the corresponding symbols where the CP is present.

We now show that it is indeed possible to have  $\|\gamma_m\|_\infty \rightarrow 0$  in the interior of a large enough frame i.e. when  $D \leq m \leq N - D$  and  $N$  is large if the poles of a certain inverse filter are not on the unit circle. Let us write

$$\tilde{g}_n = [H(z)H^\dagger(1/z^*) + K]^{-1} H^\dagger(1/z^*), \quad (24)$$

where  $H(z)$  is matrix function of scalar variable  $z$  given by

$$H(z) = \sum_{l=0}^{L-1} h_l z^l, \quad (25)$$

$z = e^{-j2\pi n/N}$ ,  $K = 0$  for the ZF SC-FDE, and  $K = N_0$  for the MMSE SC-FDE. Note that for  $z \neq 0$ , we can write

$$\tilde{g}_n = [z^{L-1}H(z)H^\dagger(1/z^*) + Kz^{L-1}]^{-1} z^{L-1}H^\dagger(1/z^*). \quad (26)$$

Let us define

$$R(z) \triangleq \det(z^{L-1}H(z)H^\dagger(1/z^*) + Kz^{L-1}). \quad (27)$$

We note that  $R(z)$  is self-reciprocal since

$$R(z) = z^{2L-2} R^*(1/z^*). \quad (28)$$

As a consequence if  $\phi$  ( $|\phi| \neq 0$ ) is a root of  $R(z)$  i.e.  $R(\phi) = 0$ , then  $R(\phi) = \phi^{2L-2} R^*(1/\phi^*) = 0$ , or  $1/\phi^*$  is also a root of  $R(z)$ . Hence we can write

$$R(z) = c \prod_{k=1}^{N_r(2L-2)} (z - \beta_k)(z - 1/\beta_k^*), \quad (29)$$

where  $c$  is a constant dependent on the channel. Note that any element of  $\tilde{g}_n$  in (26) will have  $R(z)$  in the denominator, and furthermore, the degree of the numerator (i.e. the highest power of  $z$ ) will be smaller than  $R(z)$ . Hence the partial fraction expansion of the  $(i, j)$  element of  $\tilde{g}_n$  can be written as

$$\tilde{g}_n^{i,j} = \sum_{l=1}^P \frac{\alpha_l^{i,j}}{z - \beta_l}, \quad (30)$$

where  $z = e^{-j2\pi n/N}$ ,  $P = 2N_r(L-1)$  for  $N_r > 1$  and  $P = L-1$  for  $N_r = 1$  for the ZF SC-FDE, and we have assumed that all the  $\beta_l$ 's are distinct, an event that occurs with probability 1 for a stochastic fading channel. Then using (21), we have the following expression for the  $(i, j)$ th element of  $\gamma_m$

$$\begin{aligned}\gamma_m^{i,j} &= \frac{1}{N} \sum_{l=1}^P \sum_{n=0}^{N-1} \frac{\alpha_l^{i,j}}{e^{-j2\pi n/N} - \beta_l} e^{j2\pi mn/N} \\ &= \sum_{l=1}^P \alpha_l^{i,j} \sum_{n=0}^{N-1} \frac{e^{j2\pi mn/N}}{N} \sum_{q=0}^{\infty} \left[ e^{j2\pi(q+1)n/N} \beta_l^q \mathbf{1}_{|\beta_l|<1} - \frac{e^{-j2\pi qn/N}}{(\beta_l)^{q+1}} \mathbf{1}_{|\beta_l|>1} \right]\end{aligned}\quad (31)$$

$$= \sum_{l=1}^P \alpha_l^{i,j} \sum_{q=0}^{\infty} \frac{1}{N} \sum_{n=0}^{N-1} \left[ e^{j2\pi(m+q+1)n/N} \beta_l^q \mathbf{1}_{|\beta_l|<1} - \frac{e^{j2\pi(m-q)n/N}}{(\beta_l)^{q+1}} \mathbf{1}_{|\beta_l|>1} \right]\quad (32)$$

$$= \sum_{l=1}^P \alpha_l^{i,j} \sum_{q=0}^{\infty} \left[ \delta_{q,(rN+N-1-m)} \beta_l^q \mathbf{1}_{|\beta_l|<1} - \delta_{q,(m+rN)} \mathbf{1}_{|\beta_l|>1} \right]\quad (33)$$

$$= \sum_{l=1}^P \alpha_l^{i,j} \left[ \beta_l^{N-1-m} \sum_{r=0}^{\infty} \beta_l^r \mathbf{1}_{|\beta_l|<1} - \frac{1}{(\beta_l)^{m+1}} \sum_{r=0}^{\infty} (\beta_l)^{-rN} \mathbf{1}_{|\beta_l|>1} \right]\quad (34)$$

$$= \sum_{l=1}^P \alpha_l^{i,j} \left[ \frac{\beta_l^{N-1-m}}{1 - \beta_l^N} \mathbf{1}_{|\beta_l|<1} - \frac{1}{(\beta_l)^{m+1}} \frac{1}{[1 - (\beta_l)^{-N}]} \mathbf{1}_{|\beta_l|>1} \right]\quad (35)$$

$$= \sum_{l=1}^P \alpha_l^{i,j} \frac{\beta_l^{N-1-m}}{1 - \beta_l^N},\quad (36)$$

where in (33),  $\delta_{i,j} = 1$  if  $i = j$  and is zero otherwise,  $\mathbf{1}_{\text{condition}}$  is the indicator function that is 1 when the 'condition' is true and is zero otherwise. It follows from the above expression that the contribution of the pole  $\beta_l$  to the 'Edge Effect' decreases exponentially as  $\beta_l^{-m-1}$  as one moves inside the frame from its head for  $|\beta_l| > 1$ , and as  $\beta_l^{N-m-1}$  as one moves inside the frame from its tail for  $|\beta_l| < 1$ . For the case of MMSE SC-FDE where the roots occur in conjugate reciprocal pairs, the 'Edge Effect' is determined primarily by the pair of roots closest to the unit circle that make it decay with the same rate from both the head and the tail of the frame.

We also note that for  $K > 0$ ,

$$R(e^{jw}) = e^{jw(L-1)} \det \left( H(e^{jw}) H^\dagger(e^{jw}) + KI \right) > 0, \quad \forall \quad w. \quad (37)$$

Hence for the MMSE SC-FDE, there is no pole on the unit circle and the edge effect will decay exponentially as one moves within the frame.



#### D. CP-less OFDM

For the OFDM transmission, one can recover the symbol tones by taking the DFT of the equalized signal recovered without the CP as in the previous section. The SINR expression unlike the SC-FDE, doesn't have a closed form expression.

#### IV. CASE OF A TWO-PATH FADING CHANNEL

Let us consider the case of a two tap fading channel for  $N_r = N_t = 1$  whose  $z$ -transform is given by

$$H(z) = h_0 + h_1 z^d, \quad (38)$$

where  $d$  is an integer that denotes the delay of the second path, and  $h_0$  and  $h_1$  are  $\mathcal{CN}(0, 1)$ . For the case of ZF SC-FDE, we look at the roots of  $H(z)$  that have a magnitude of  $|h_0/h_1|^{1/d}$ . The probability that there are roots that have the magnitude in the interval  $(1 - \epsilon, 1 + \epsilon)$ , with  $\epsilon \in (0, 1)$ , is given by

$$p_\epsilon = \mathbb{E} \left\{ \text{Prob} \left[ |h_0|^{2/d} \in (|h_1|^{2/d}(1 - \epsilon)^2, |h_1|^{2/d}(1 + \epsilon)^2 |h_1|) \right] \right\} \quad (39)$$

$$= \frac{(1 + \epsilon)^{2d} - (1 - \epsilon)^{2d}}{1 + (1 - \epsilon)^{2d} + (1 + \epsilon)^{2d} + (1 - \epsilon^2)^{2d}}. \quad (40)$$

For  $d = 1$ , this simplifies to

$$p_\epsilon = \frac{\epsilon}{1 + \epsilon^4/4}. \quad (41)$$

For small  $\epsilon$ ,  $p_\epsilon \approx d\epsilon$ .  $p_\epsilon$  increases with  $d$  since it makes the roots of  $H(z)$  to be closer to the unit circle. This implies that the response of the inverse filter (see (21)) will take longer to die down.

For the case of the MMSE SC-FDE, we need to look at the roots of

$$\begin{aligned} R(z) &= z^d H(z) H^* \left( \frac{1}{z^*} \right) + K z^d \\ &= h_0 h_1^* + (|h_0|^2 + |h_1|^2 + K) z^d + h_0^* h_1 z^{2d}. \end{aligned}$$

The roots of  $R(z)$  are the  $d$ th roots of

$$\rho_{1,2} = \frac{-\psi \pm \sqrt{\psi^2 - 4|h_0|^2|h_1|^2}}{2h_0^* h_1}, \quad (42)$$

where  $\psi = |h_0|^2 + |h_1|^2 + K$ . It is easily shown that  $\rho_1 = 1/\rho_2^*$ ,  $|\rho_2| > |\rho_1|$ , and hence  $|\rho_2| \geq 1$ , and

$$|\rho_2| \geq \max \left( \left| \frac{h_0}{h_1} \right|, \left| \frac{h_1}{h_0} \right| \right), \quad (43)$$

$$|\rho_1| \leq \min \left( \left| \frac{h_0}{h_1} \right|, \left| \frac{h_1}{h_0} \right| \right). \quad (44)$$

This implies that the roots of  $R(z)$  are farther away from the unit circle as compared to the roots of  $H(z)$ , and hence the response of the inverse filter in (21) dies faster for the MMSE SC-FDE than the ZF SC-FDE.

One can simplify (42) to get

$$|h_0|^2 - \left( |\rho_2| + \frac{1}{|\rho_2|} \right) |h_0||h_1| + (|h_1|^2 + K) = 0, \quad (45)$$

which admits a solution for  $|h_0|$  only if

$$|h_1|^2 \geq \frac{4|\rho_2|^2 K}{(1 - |\rho_2|^2)^2}. \quad (46)$$

For  $|\rho_2| = 1 + \epsilon$ , where  $\epsilon > 0$ , this amounts to

$$|h_1|^2 \geq \left( \frac{1 + \epsilon}{1 + \epsilon/2} \right)^2 \frac{K}{\epsilon^2} > \frac{K}{\epsilon^2}, \quad (47)$$

an event which occurs with the probability of less than  $e^{-K/\epsilon^2}$ . For  $\epsilon < 0.5\sqrt{K}$  and  $\epsilon < 0.25\sqrt{K}$ , the probability is less than 2% and 0.000012% respectively. Hence there are no roots of  $R(z)$  in  $((1 + 0.25\sqrt{K})^{-1/d}, (1 + 0.25\sqrt{K})^{1/d})$  with probability approaching unity. Hence one can be sure with probability approaching unity that the response of the inverse filter in (21) decays exponentially with the exponent of at least  $(1 + 0.25\sqrt{K})^{1/d}$  from the head and the tail. Hence one can choose  $D$  large enough such that  $\xi_m$  in (20) or more conveniently the upper bound to  $|\xi_m|$  in (23) is small.

If the second path is weaker than the first path i.e. has an average power of  $\sigma^2$  where  $\sigma^2 < 1$ , then there are no roots of  $R(z)$  in  $((1 + 0.25\sqrt{K}/\sigma)^{-1/d}, (1 + 0.25\sqrt{K}/\sigma)^{1/d})$  with probability less than 0.000012%. This makes the response of the inverse filter decrease faster than the case when both paths have same average power. Note that if the second path has larger average power than the first path, then we can view (45) as a quadratic equation in  $|h_1|^2$  and follow the same arguments to get to the same conclusion as above.

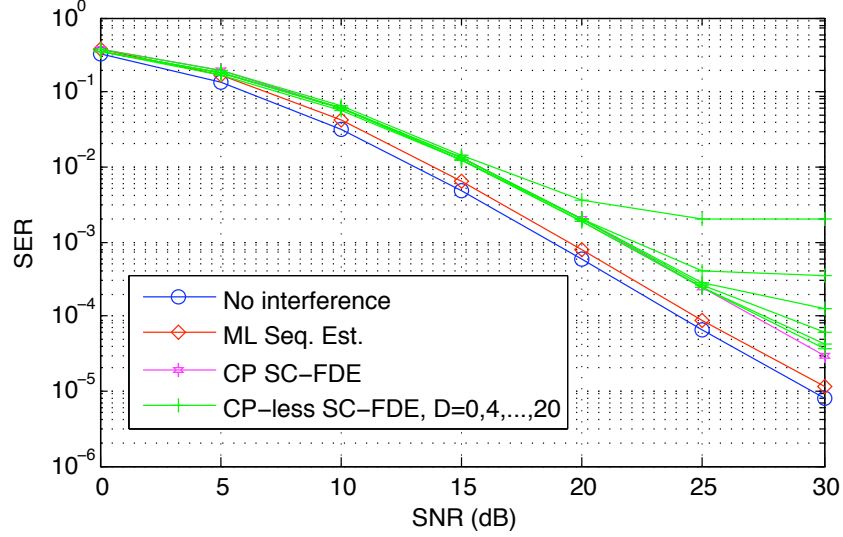


Fig. 1. Symbol Error Rate (SER) versus SNR curves for QPSK for the two-path fading channel with  $d = 1$ ,  $D = 0, 4, 8, 12, 16, 20$ , and DFT frame size of 512.

We plot the numerical results for this channel for QPSK modulation in Fig. 1 with maximum likelihood sequence estimation using the Viterbi algorithm and the SC-FDE with or without CP. We also plot the case when there is no self-interference i.e. a flat fading channel with the same average received power of unity as the ISI channel. The size of the DFT frame is 512. For the case of SC-FDE with no CP, we consider the inner portion (quantized by parameter  $D$ ) as equalized in one frame and the exterior portion of the frame from both ends forms the interior portion of some other frame. As one can see in Fig. 1, as  $D$  increases, the performance of CP-less SC-FDE approaches that of CP SC-FDE. This matches well with the earlier analytical observation that the SINR for the inner portion of the frame without CP approaches that of the SINR with CP for SC-FDE.

## V. CONCLUSIONS

It is shown that for SC-FDE without the cyclic prefix by increasing the length of the DFT and by discarding the symbols on either ends of the DFT frame one can asymptotically obtain the same value of SINR as the case when the cyclic prefix is present. We showed for a two-path fading channel (whose inverse channel has a long tail) that the proposed method presents a method to do away with the redundancy due to the cyclic prefix by a moderate increase in the

receiver complexity. The loss of redundancy could be used to achieve high data rates due to increase in the number of channel uses.

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## APPENDIX

We can write using (11)

$$\tilde{\mathcal{R}} = \sqrt{N}\tilde{\mathcal{H}}\mathcal{F}^\dagger\mathcal{Y} + \tilde{\mathcal{V}}, \quad (48)$$

where  $\tilde{\mathcal{R}} = [\tilde{r}_0 \cdots \tilde{r}_{N-1}]^T$ ,  $\mathcal{F}$  is a  $NN_t \times NN_t$  block diagonal unitary matrix given by  $\mathcal{F} = \text{diag}[F, \cdots, F]$ , where  $F$  is the  $N \times N$  DFT unitary matrix,  $\tilde{\mathcal{Y}} = [\tilde{y}_0 \cdots \tilde{y}_{N-1}]^T$ , and  $\tilde{\mathcal{V}} = [\tilde{v}_0 \cdots \tilde{v}_{N-1}]^T$ . Since we are interested in obtaining  $Y$ , we can write the ZF or MMSE pre-multiplying matrix as (with  $K = 0$  for ZF and  $K = N_0$  for MMSE)

$$\begin{aligned} \mathcal{G} &= \sqrt{N} \left( N\mathcal{F}\tilde{\mathcal{H}}^\dagger\tilde{\mathcal{H}}\mathcal{F}^\dagger + KI \right)^{-1} \mathcal{F}\tilde{\mathcal{H}}^\dagger \\ &= \sqrt{N}\mathcal{F} \left( N\tilde{\mathcal{H}}^\dagger\tilde{\mathcal{H}} + KI \right)^{-1} \tilde{\mathcal{H}}^\dagger, \end{aligned}$$

which is a concatenation of ZF or MMSE in the frequency domain followed by DFT. Note that  $\mathcal{G} = \text{diag}[g_0, \cdots, g_{N-1}]$  is a block diagonal matrix, where

$$\tilde{g}_n = \sqrt{N}(N\tilde{h}_n^\dagger\tilde{h}_n + KI)^{-1}\tilde{h}_n^\dagger. \quad (49)$$